RADIOMETRY

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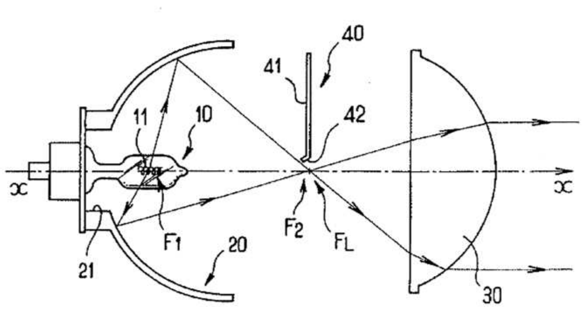


FIG. 2

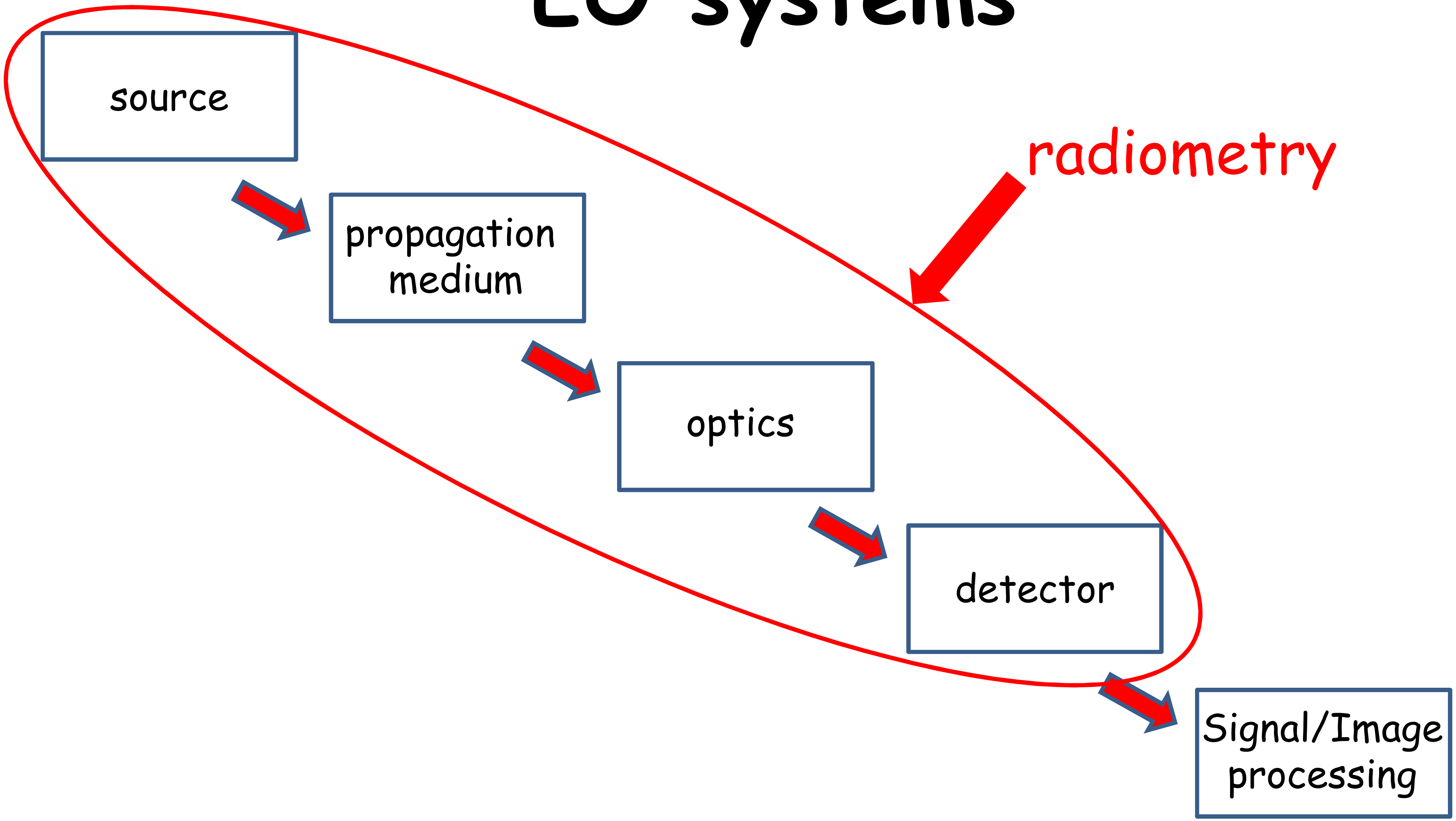
Summary

- Introduction
- Fundamental quantities
- Laws of conservation
- Relationships between fundamental quantities
  - in free space
  - in optical instruments
- The blackbody
- Natural sources

General definitions

- Radiometry = theoretical and experimental characterization of electromagnetic radiations :
  - fundamental quantities associated with these radiations
  - relationships between these quantities
  - instrumentation to make characterizations
- Photometry = radiometry for the visible spectrum

Radiometric characterization of EO systems



FUNDAMENTAL QUANTITIES

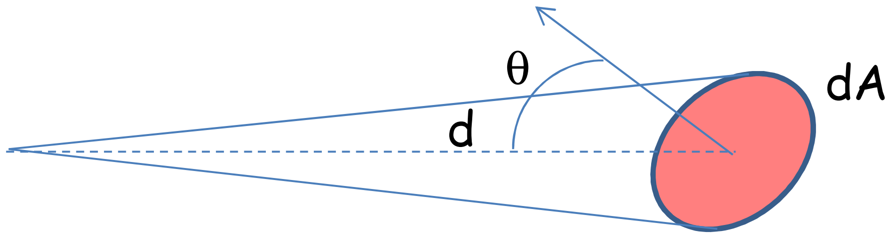
Flux F

- Definition : optical radiation rate
- 3 units :
  - energetic flux F_e in watts
  - photon flux F_p in s^{-1}
  - visual flux F_v in lm = lumens

Source	Power supply (W)	F_e (W)	F_p (s^{-1})	F_v (lm)
Tungsten lamp	100	90	10^{21}	1500
Nd:YAG laser	20	1	$5\ 10^{18}$	0

Solid angle

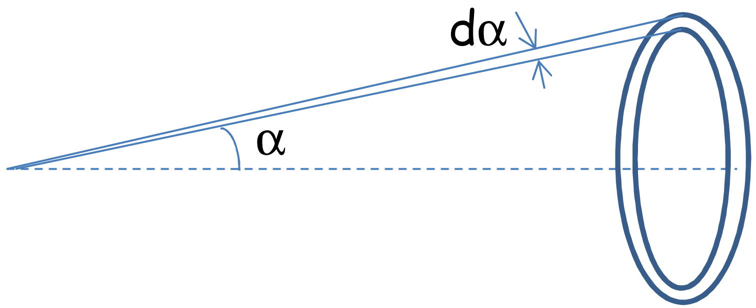
- Small planar object :



obliquity

$$d\Omega = \frac{dA \cos\theta}{d^2}$$

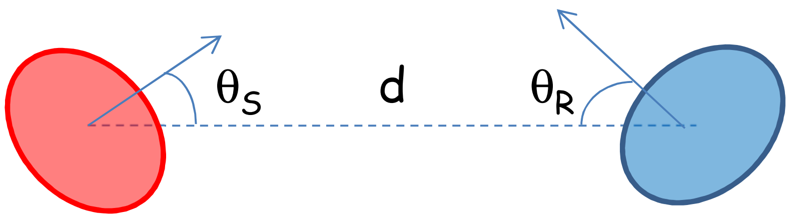
- Circular on axis ring :



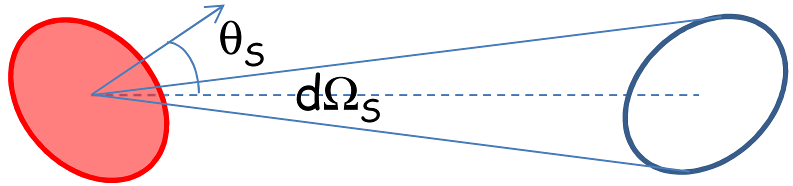
$$d\Omega = 2\pi \sin\alpha \, d\alpha$$

Geometrical extent or étendue

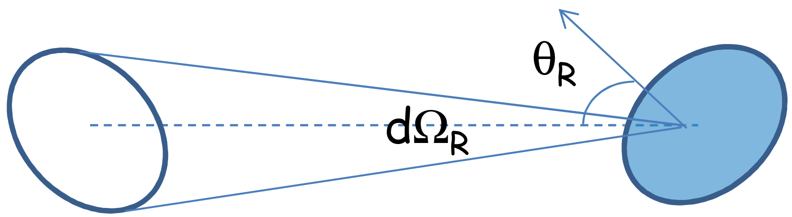
- Case of a pencil of light
  - A pencil of light is defined either by two small apertures at finite distance from each other, or by one a unique aperture and a small solid angle



$$d^2G = \frac{dA_S \cos\theta_S \, dA_R \cos\theta_R}{d^2}$$



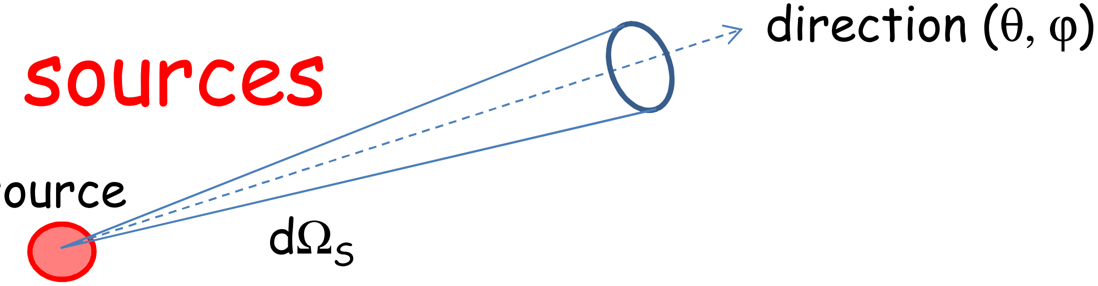
$$d^2G = dA_S \cos\theta_S \, d\Omega_S$$



$$d^2G = dA_R \cos\theta_R \, d\Omega_R$$

Intensity

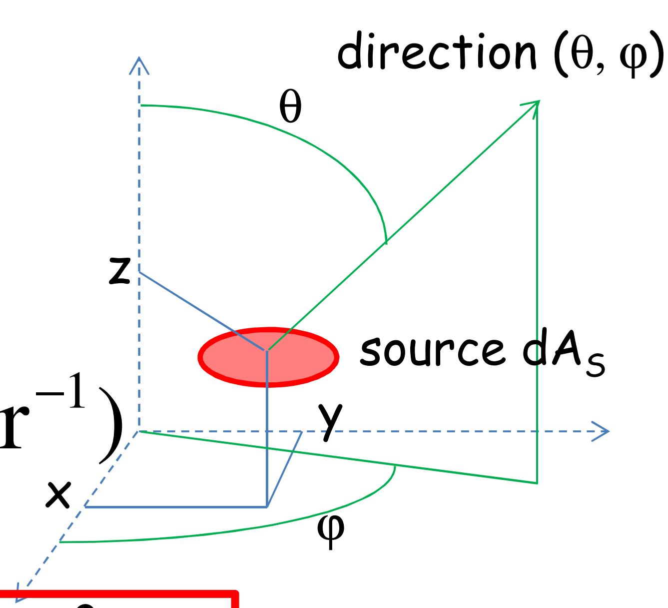
- For unresolved sources



$$I(\theta,\phi)=\frac{dF_S}{d\Omega_S}\quad (W\ \ sr^{-1})$$

- Intensity diagram : $I(\theta,\varphi)$
- Isotropic source : $I(\theta,\varphi)=cte$

Radiance



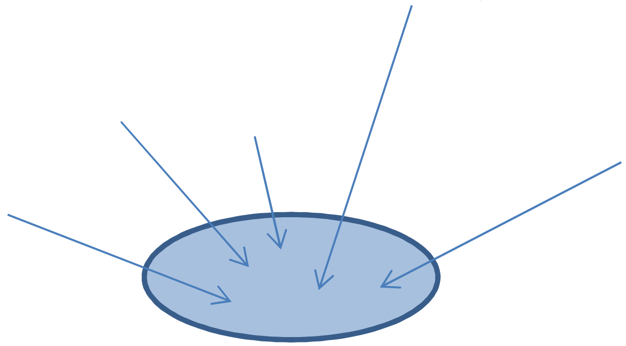
$$L(x,y,z,\theta,\phi) = \boxed{\frac{dI}{dA_{S,app}}} \left(W\ m^{-2}\ sr^{-1}\right)$$

$$= \frac{d^2F_S}{d\Omega_S dA_S \cos\theta_S} = \boxed{\frac{d^2F_S}{d^2G}} \left(W / m^2 sr\right)$$

$\Rightarrow$ (fundamental law, for a pencil) $\boxed{d^2F_S = L\ d^2G}$

$If\ L(x,y,z,\theta,\varphi) = cte \Rightarrow$ the source is <u>lambertian</u>

Irradiance



$$E(x',y',z') = \frac{dF_R}{dA_R} \quad (W\ m^{-2})$$

Rq : visible apparent magnitude m $= -2.5 \log \frac{E}{2.87\ 10^{-8}\ W/m^2}$

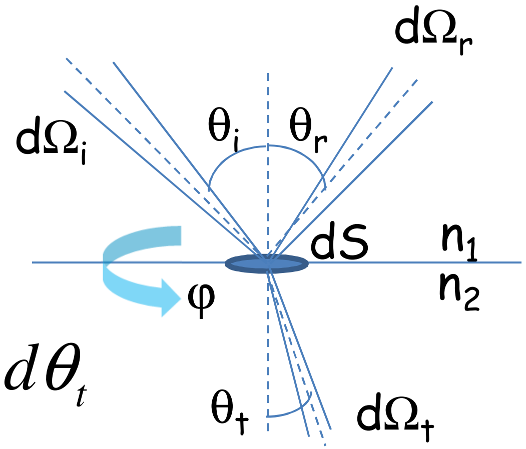
Systèmes d'unités, vocabulaire

Grandeur	Unités énergétiques	Unités photoniques	Unités lumineuses
Flux	W (Power)	s-1	lumen (lm) (Flux)
Intensité	W sr-1 (Intensity)	s-1 sr-1	candela (cd) (Intensity)
Luminance	W m-2 sr-1 (Radiance)	s-1 m-2 sr-1	cd m-2 (Luminance)
Exitance	W m-2 (Exitance)	s-1 m-2	lm m-2
Éclairement	W m-2 (Irradiance)	s-1 m-2	lux (Illuminance)
Quantité de lumière	J	nombre de photons	lm s
Exposition	J m-2 (Fluence)	Nb de photons m-2	lux s

LAWS OF CONSERVATION

Conservation of (optical) étendue

- In free space : $d^2G = \dfrac{dA_S \cos\theta_S dA_R \cos\theta_R}{d^2} = cte$
- At refraction :



$$(1)\ d^2G_i = dS\cos\theta_i d\Omega_i = dS\cos\theta_i \sin\theta_i d\theta_i d\varphi$$

$$(2)\ d^2G_t = dS\cos\theta_t d\Omega_t = dS\cos\theta_t \sin\theta_t d\theta_t d\varphi$$

$$(3)\ n_1 \sin\theta_i = n_2 \sin\theta_t \Rightarrow (4)\ n_1 \cos\theta_i d\theta_i = n_2 \cos\theta_t d\theta_t$$

$$(1), (2), (3), (4) \Rightarrow \boxed{n_1^2 d^2G_i = n_2^2 d^2G_t}$$

- At reflexion : $\boxed{d^2G_i = d^2G_r}$
- In optical systems : the étendue is constant from object (in air) to image (in air)

Conservation of radiance

In the vacuum :

$$d^2F_R = d^2F_S \Rightarrow L_R d^2G_R = L_S d^2G_S \Rightarrow \boxed{L_R = L_S}$$

For the case of reflexion :

$$d^2F_r = Rd^2F_i \Rightarrow L_r d^2G_r = RL_i d^2G_i \Rightarrow \boxed{L_r = RL_i}$$

For the case of refraction :

$$d^2F_t = Td^2F_i \Rightarrow L_t d^2G_t = TL_i d^2G_i \Rightarrow \boxed{\frac{L_t}{n_t^2} = T\frac{L_i}{n_i^2}}$$

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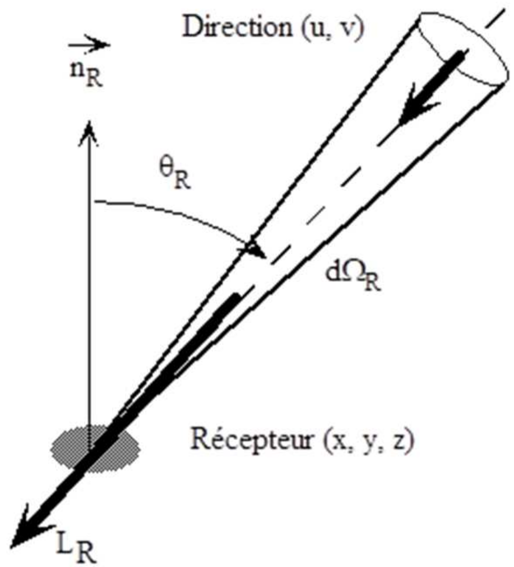
RELATIONS BETWEEN FUNDAMENTALS QUANTITIES IN FREE SPACE

Relations between fundamentals quantities in free space

- Intensity → flux
- Radiance → flux
- Radiance → intensity
- Radiance → irradiance
  - Solar constant
- Intensity → irradiance
  - Law of Bouguer
- …

Relation between radiance and irradiance



Pencil of light :

$$d^2F_R = Ld^2G = LdA_R \cos\theta_R d\Omega_R$$

$$\Rightarrow dE_R = L\cos\theta_R d\Omega_R$$

Solar constant :

Sun

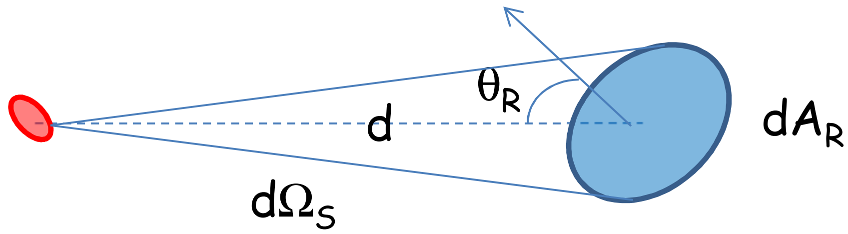
* $\approx$ small disc, angular radius $\alpha_S \approx 15$ arcmin

* total energetic radiance $\approx 2.2\ 10^7\ \mathrm{W\ m^{-2}\ sr^{-1}}$

In space, in a plane perpendicular to the rays :

$$\Delta E_R = L\ \Delta\Omega_R = L\ \pi\alpha_S{}^2 \approx 1400\ \mathrm{W\ m^{-2}}$$

Bouguer*'s law



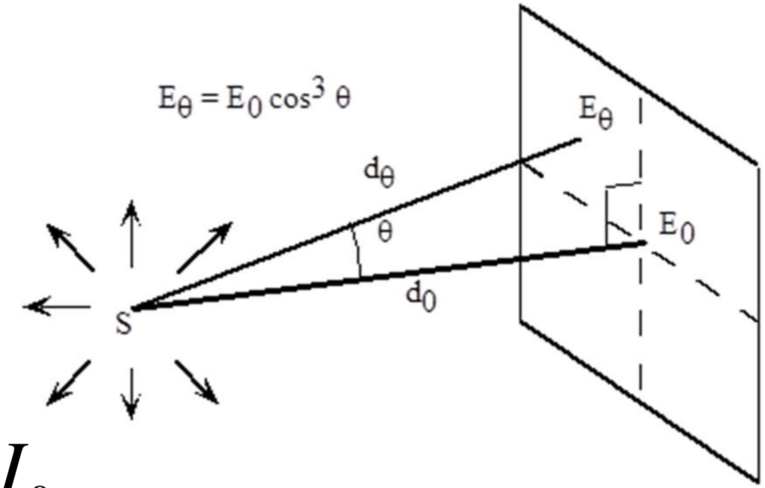
Flux incident upon elementary area dA_r :

$$dF_R = I(\theta,\phi)\, d\Omega_S \underset{\substack{d>>1 \\ \text{or } dA_R <<1}}{=} I(\theta,\phi)\frac{dA_R \cos\theta_R}{d^2}$$

$\Rightarrow$ resulting irradiance $E = \dfrac{dF_R}{dA_R} = \dfrac{I(\theta,\phi)\cos\theta_R}{d^2}$

* 1698-1758, géophysicien français, fondateur de la photométrie

Bouguer's law :
case of the isotropic intensity point source

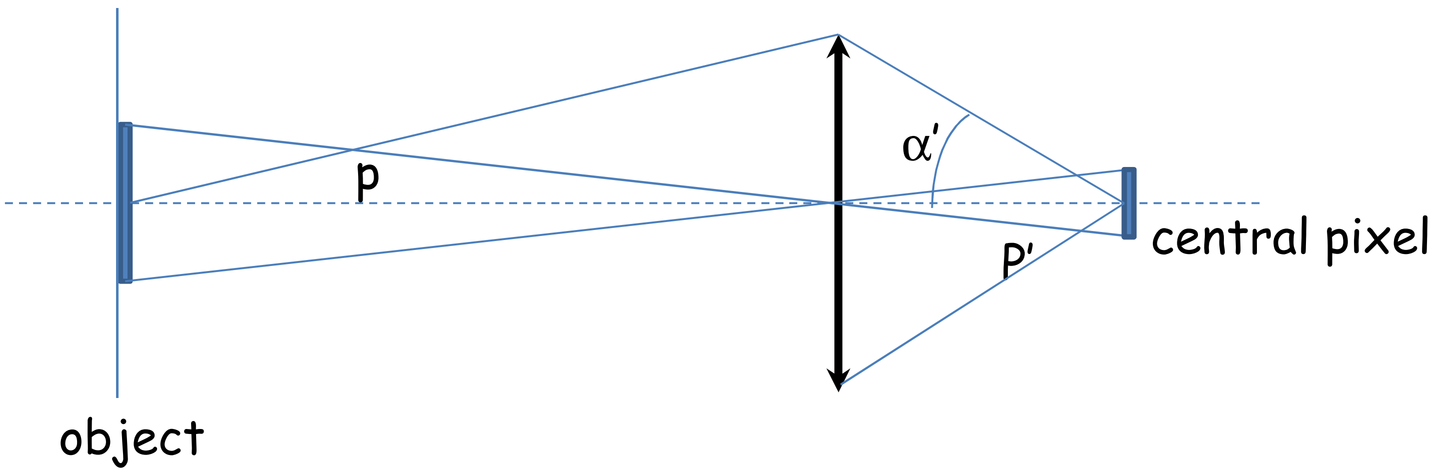


$$E_0 = \frac{I_0}{{d_0}^2}$$

$$E_\theta = \frac{I_\theta \cos\theta}{{d_\theta}^2} = \frac{I_0 \ \cos\theta}{\left(\dfrac{d_0}{\cos\theta}\right)^2} = E_0 \cos^3\theta$$

RELATIONS IN THE INSTRUMENTS

Imagery systems – on axis



Geometric extent for the central pixel :

- in image space : $G \underset{\infty/F}{=} S_{op} \dfrac{A_R}{f^2}$

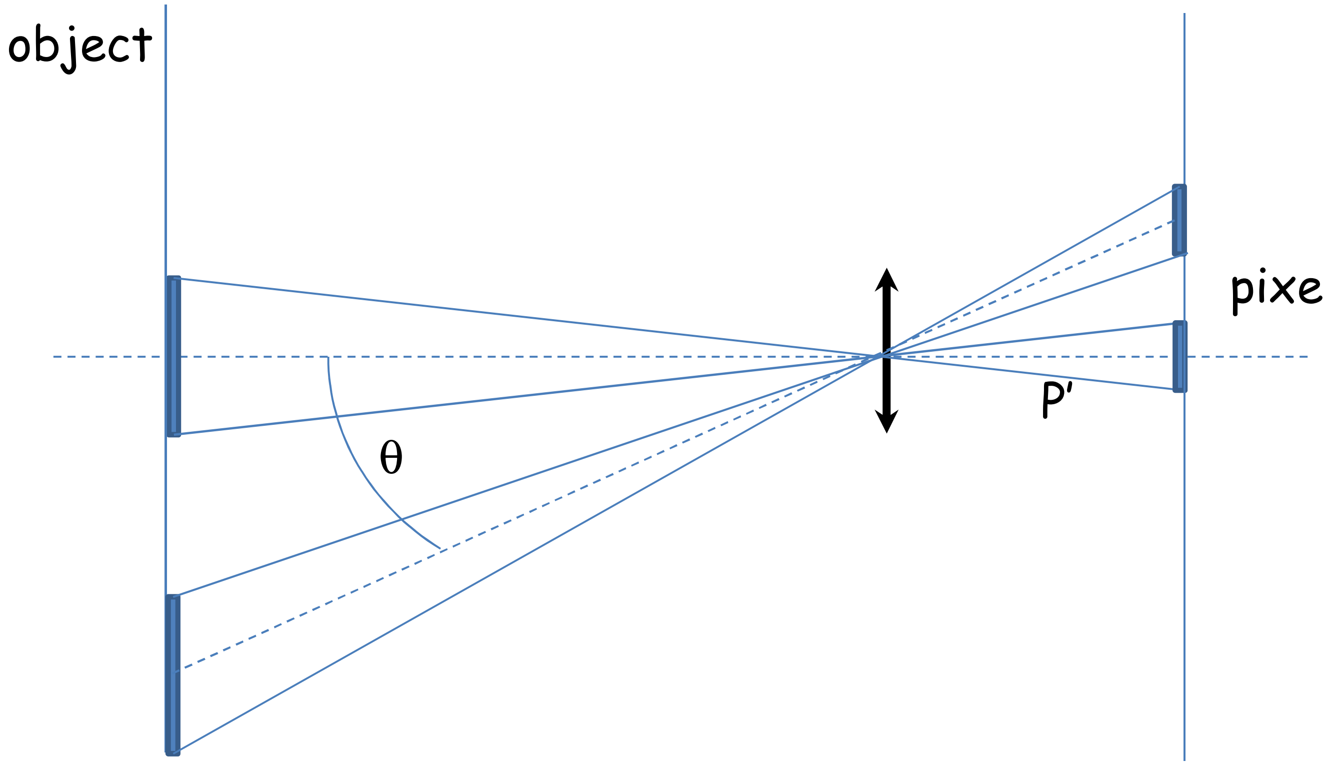
Imagery systems – on axis

Received flux for the central pixel :

$$F_R = L_R G = T\ T_{op}\ \ L_S\ G$$

$$\Rightarrow F_R \underset{\infty/F}{=} \pi\ T\ T_{op}\ \ L_S\ S_{op}\ \frac{A_R}{f^2}$$

Off axis : $\cos^4\theta$ law



Off axis : $\cos^4\theta$ law

Assumptions : low aperture, single lens system, radiance is uniform in object space :

$$G_0 \approx \frac{S_{op}\ A_d}{p'^2}$$

$$G_\theta \approx \frac{\left(S_{op}\ \cos\theta\right)\ \left(A_d\ \cos\theta\right)}{\left(\dfrac{p'}{\cos\theta}\right)^2} = G_0 \cos^4\theta$$

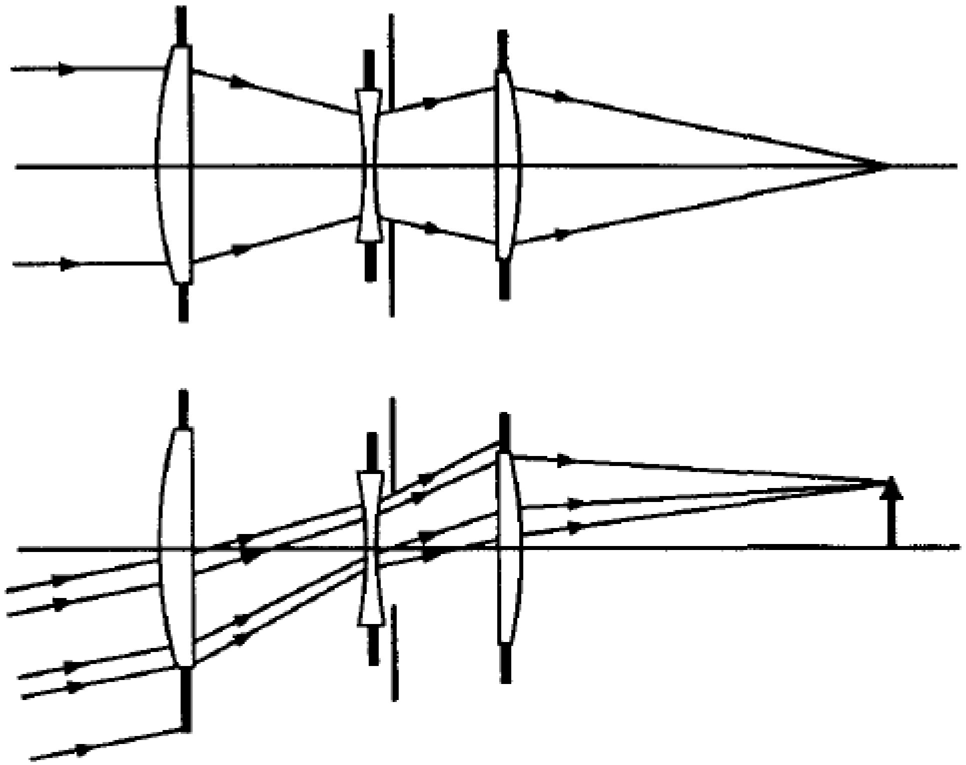
$$\underset{L=cte}{\Rightarrow} \boxed{F_\theta = F_0 \cos^4\theta}$$

$\cos^4\theta$ law

example of « flat field »

Off axis : vignetting

Case of multi-element systems



Optical system design, R. Fisher

THE BLACKBODY

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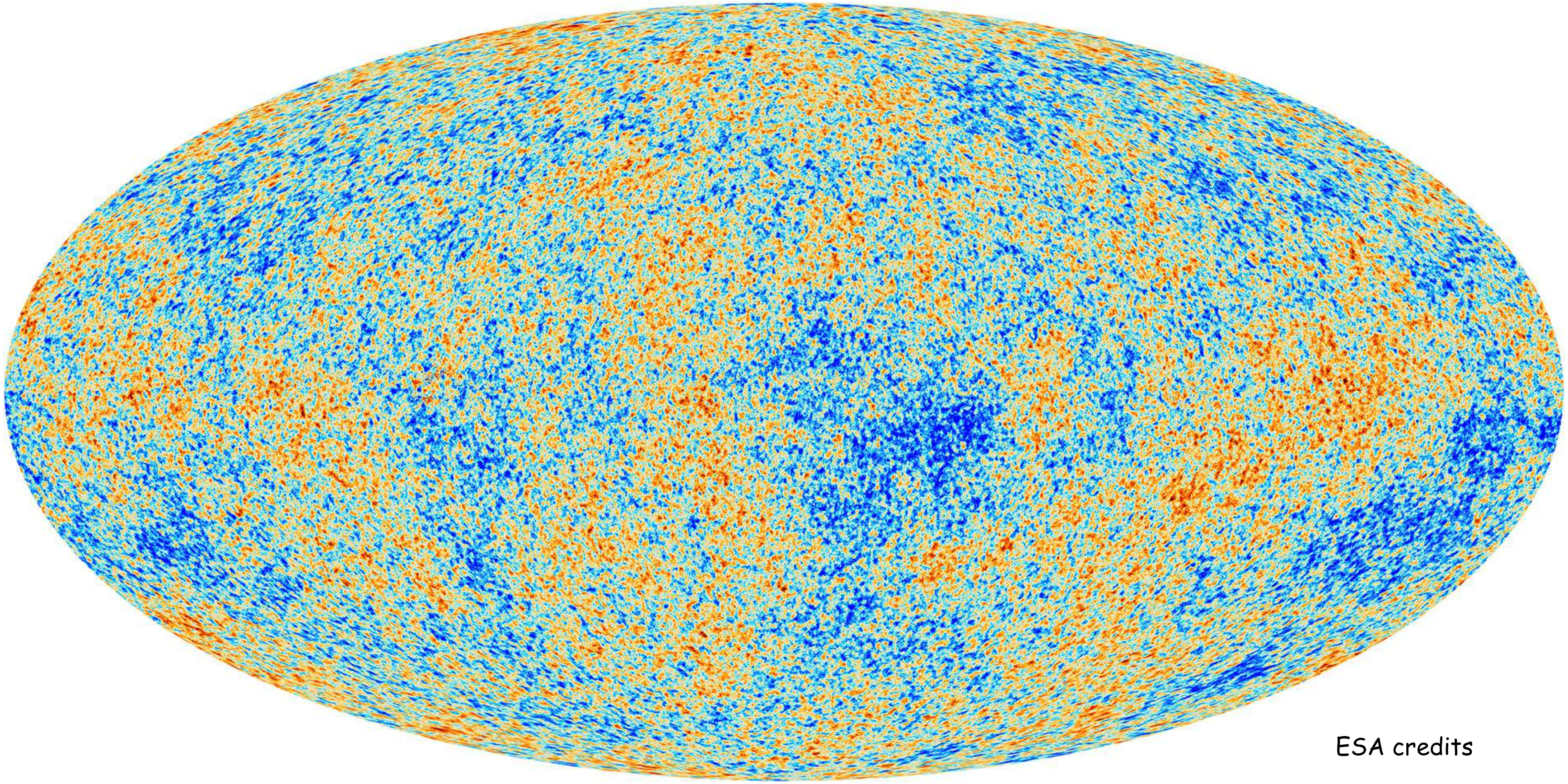
Seurat's painting ?



ESA credits

Kirchhoff's law

Definition : Y is a blackbody, ie.

$$A_Y(\lambda, \theta_Y, \varphi_Y, polar, T) \underset{\forall \lambda, \theta_Y, \varphi_Y, polar, T}{=} 1,$$

Thus Kirchhoff's law 1 :

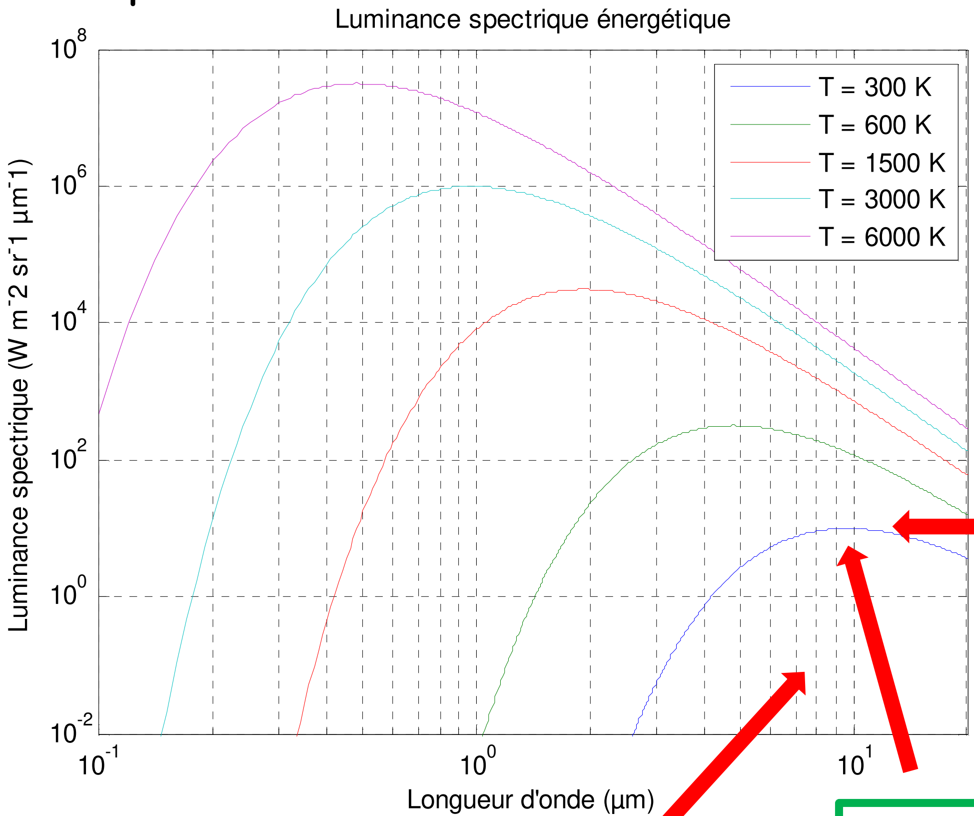
$$\left[\frac{dL_e}{d\lambda}\right]_X^T (\lambda, \theta_X, \varphi_X) = A_X(\lambda, \theta_X, \varphi_X, polar, T)\left[\frac{dL_e}{d\lambda}\right]_{BB}^T (\lambda, \theta_{BB}, \varphi_{BB})$$

So, at a given temperature, the blackbody has the highest spectral radiance.

Blackbody radiation laws

(energetic or radiant units)

Spectral radiance



Planck's law

$$\left[\frac{dL_e}{d\lambda}\right]_{BB}^{T} = \frac{2\,h\,c^2}{\lambda^5}\,\frac{1}{e^{\frac{h\,c}{\lambda\,k\,T}}-1}$$

$$\left[\frac{dL_e}{d\lambda}\right]_{BB}^{T}(\lambda_m) = K_2 T^5$$

$$K_2 = 4.093\,10^{-12}\,Wm^{-2}sr^{-1}\mu m^{-1}K^{-5}$$

Wien's displacement law

$$\lambda_m T = K_1$$

$$K_1 = 2898\,\mu m\,K$$

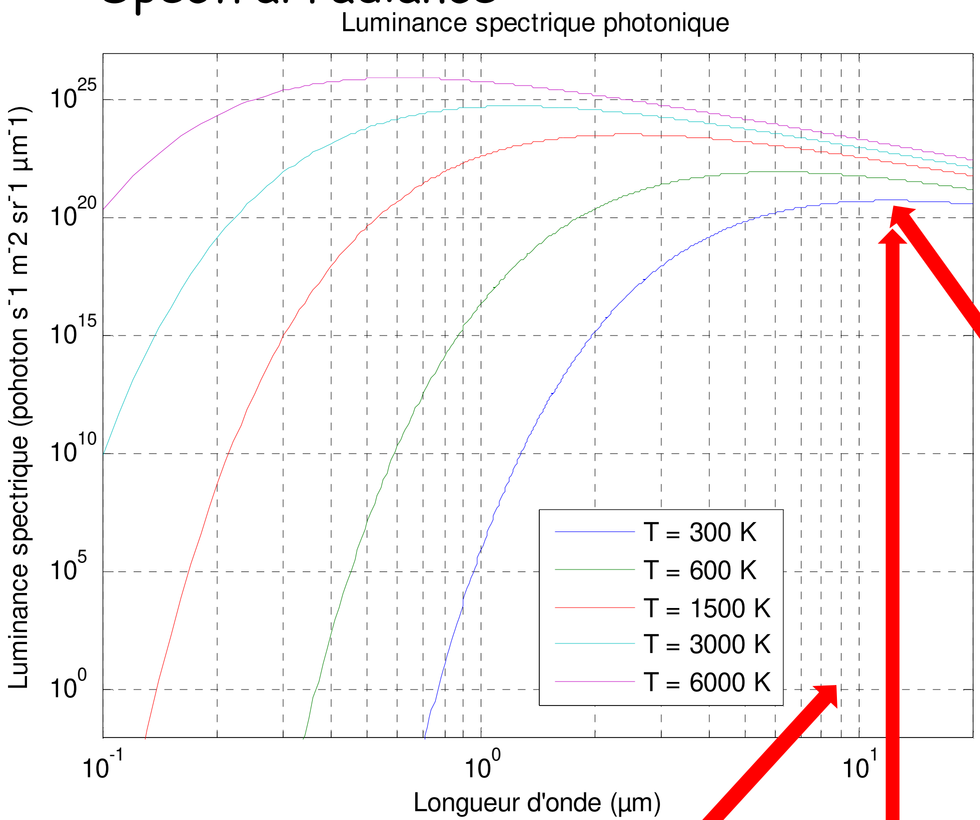
Stefan's law

$$[L_e]_{BB}^{T} = \int_0^{\infty}\left[\frac{dL_e}{d\lambda}\right]_{BB}^{T}(\lambda)\,d\lambda = K_3 T^4$$

$$K_3 = 1.804\,10^{-8}\,Wm^{-2}sr^{-1}K^{-4}$$

Blackbody radiation laws
(photonic units)

Spectral radiance



Planck's law

$$\left[\frac{dL_p}{d\lambda}\right]_{BB}^{T} = \frac{2\,c}{\lambda^4}\frac{1}{e^{\frac{h\,c}{\lambda\,k\,T}}-1}$$

$$\left[\frac{dL_p}{d\lambda}\right]_{BB}^{T}(\lambda'_m) = K'_2 T^4$$

$$K'_2 = 6.686\,10^{10}\,s^{-1}m^{-2}sr^{-1}\mu m^{-1}K^{-4}$$

Wien's displacement law

$$\lambda'_m T = K'_1$$

$$K'_1 = 3670\,\mu m\,K$$

Stefan's law

$$\left[L_p\right]_{BB}^{T} = \int_0^\infty \left[\frac{dL_p}{d\lambda}\right]_{CN}^{T}(\lambda)\,d\lambda = K'_3 T^3$$

$$K'_3 = 4.840\,10^{14}\,s^{-1}m^{-2}sr^{-1}K^{-3}$$

Properties

$$A_{BB}(\lambda,\theta_{BB},\varphi_{BB},polar,T)_{\forall\lambda,\theta_{BB},\varphi_{BB},polar,T} = 1$$

- Radiation of blackbody is lambertian
- Radiation of BB is not polarized (thermal source)
- Radiation of BB is naught at 0 K
- Two BB curves at different temperatures never cross
- Whatever the temperature (energetic unit) :
  - 25 % of radiation between 0 and λ_m
  - 75 % of radiation between λ_m and infinity
  - 98 % of radiation between 0,5 λ_m and 8 λ_m

Natural sources

- **Sun**
  - blackbody, à ~ 5900 K : λ_m ~ 0.5 μm
  - angular diameter: ~ 30'
  - solar constant = irradiance in a plane perpendicular to the rays, in space, close to the Earth : ~ 1400 W m^{-2} ou ~ 120000 lux
  - irradiance in a plane perpendicular to the rays, at the ground level (very clear weather) : ~ 900 W m^{-2} ou ~ 80000 lux
- **Sky**
  - visual radiance (= luminance) of the blue sky, on the ground, far from the direction of the sun : ~ 2000 cd m^{-2}
  - the radiation from the sky is polarized
  - white cloud : ~ lambertian, albedo > 60 %, hence a luminance close to 20000 cd m^{-2}
  - cloudy sky : luminance smaller then 1000 cd m^{-2}, hence an irradiance at ground level smaller than 3000 lux
- **Full moon**
  - in the visible : lambertian secondary source, albedo ~ 7 %, hence an apparent luminance at ground level (very clear weather) ~ 2600 cd m^{-2}
  - angular diameter: ~ 30'
  - irradiance in a plane perpendicular to the rays, at the ground level (very clear weather) : < 1 lux
- **Others**
  - Stars : irradiance in a plane perpendicular to the rays, at the ground level (very clear weather) ~ 10^{-3} lux
  - Cloudy night: irradiance in a plane perpendicular to the rays, at the ground level ~ 10^{-4} lux
  - Cloudy night, in a forest : irradiance in a plane perpendicular to the rays at the ground level ~ 10^{-5} lux